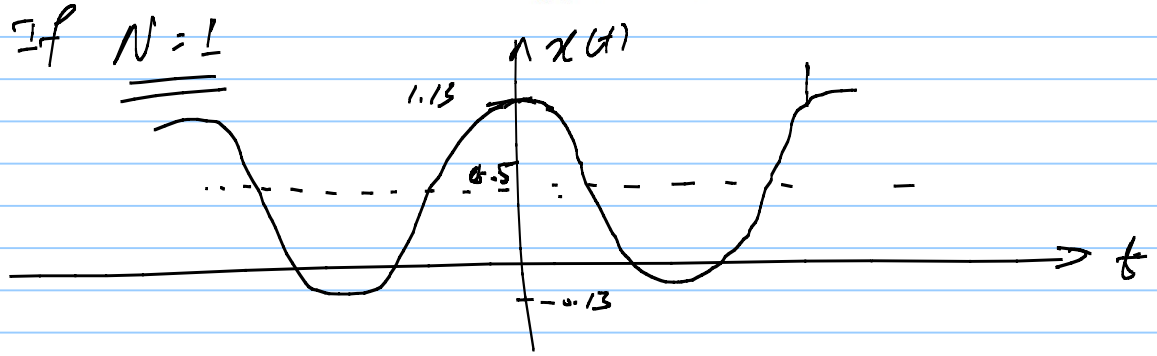
EE362 Spring 2019 Lecture Notes 10

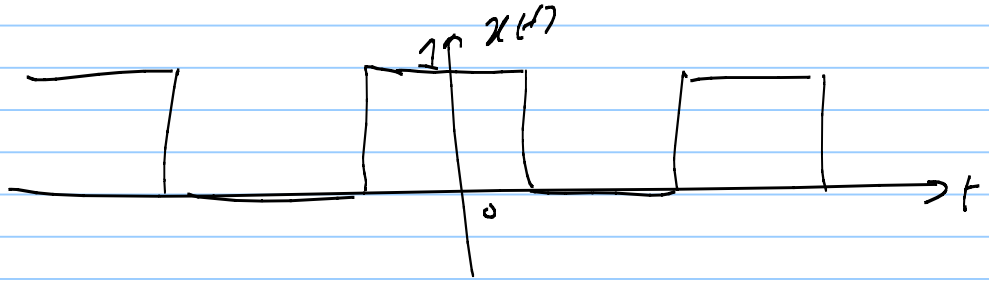
Note Title 2/26/2019

$$x(t) = \frac{1}{2} + \sum_{k=1}^{N} \left[\frac{2}{k\pi} \sin\left(\frac{k\pi}{2}\right) \cos(k\pi t) \right]$$

If $N = 1$



$x(t) = \frac{1}{2} + \frac{2}{\pi} \cos(\pi t)$, $\frac{2}{\pi} = 0.63$



If $N = 2$

$$x(t) = \frac{1}{2} + \frac{2}{\pi} \cos(\pi t) + \frac{2}{2\pi} (0) \cos(2\pi t)$$

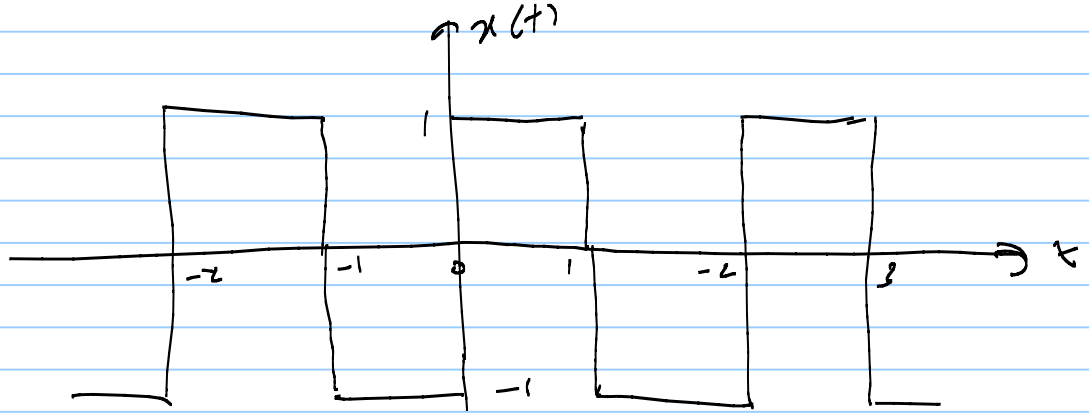
$k = 2$

If $N = 3$

$$x(t) = \frac{1}{2} + \frac{2}{\pi} \cos(\pi t) + 0 + \frac{2}{3\pi} (-1) \cos(3\pi t)$$



$x(t)=1 \;\; 0< t <1$ and $x(t)=-1 \;\; -1< t <0$ and $T=2$ (period)



Odd

$a_0 = 0 \; ; \quad a_k = 0$

$T = 2 \Rightarrow \omega_0 = \frac{2\pi}{T} = \pi$

$$b_k = \frac{4}{T}\int_0^{T/2} x(t)\sin(k\omega_0 t)\,dt$$

$$= \frac{4}{2}\int_0^{1} x(t)\sin(k\pi t)\,dt$$

$$b_k = 2\int_0^{1} (1)\sin(k\pi t)\,dt$$

$$= 2\left(\frac{-\cos(k\pi t)}{k\pi}\right)_0^{1} = \frac{-2}{k\pi}\cos(k\pi t)\Big|_0^{1}$$

$$= \frac{-2}{k\pi}\left\{\cos(k\pi) - 1\right\}$$

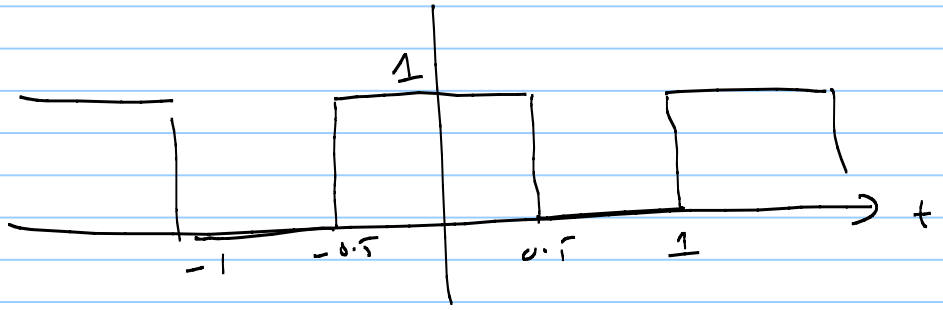
$$b_k = \frac{2}{k\pi}\left(1 - \cos(k\pi)\right)$$

If k is odd ; $b_k = \frac{4}{k\pi}$

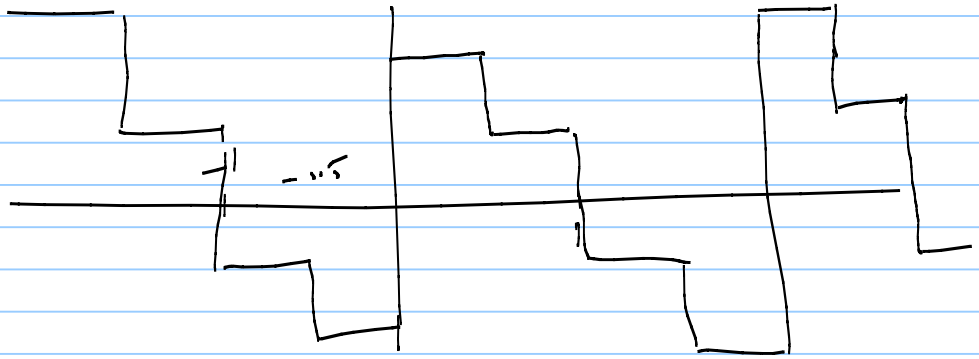
If k is even ; $b_k = 0$

$$x(t) = \sum_{k\ odd}^{\infty} \frac{4}{k\pi} \sin(k\pi t)$$

$x(t)=1$ for $-0.5< t <0.5$; and $x(t)=0$ for $-1< t <-0.5$ and $0.5< t <1$; and $T=2$ (period)



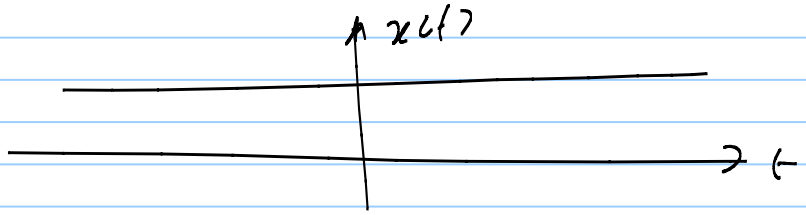
$$x(t)=\begin{cases} 1 & 0<t<0.5 \\ 0.5 & 0.5<t<1 \\ -1 & -0.5<t<0^- \\ -0.5 & -1<t<-0.5 \end{cases}$$



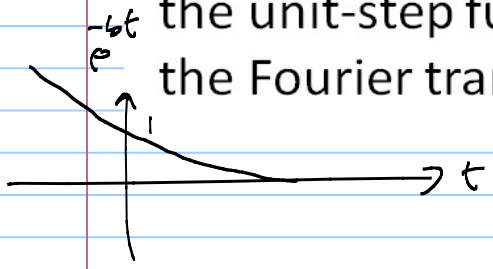
$$x(t) = 2\,e^{-j\omega t} = 2\left(\cos(\omega t) - j\sin(\omega t)\right)$$

$$X(\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \qquad -\infty \le \omega \le \infty$$

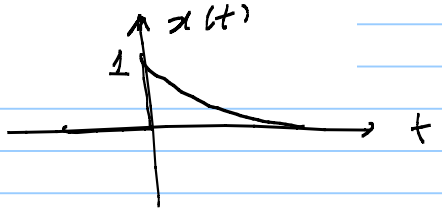
Example 1: Let $x(t)$ be a DC signal $x(t)=1$ $-\infty<t<\infty$. Since $x(t)$ is not integrable, the Fourier transform of $x(t)$ does not exist.



Example 2: Let $x(t)$ be an exponential signal, $u(t)$ is the unit-step function, and b is a real constant. Find the Fourier transform of $x(t)$.



$$x(t)=e^{-bt}u(t)$$



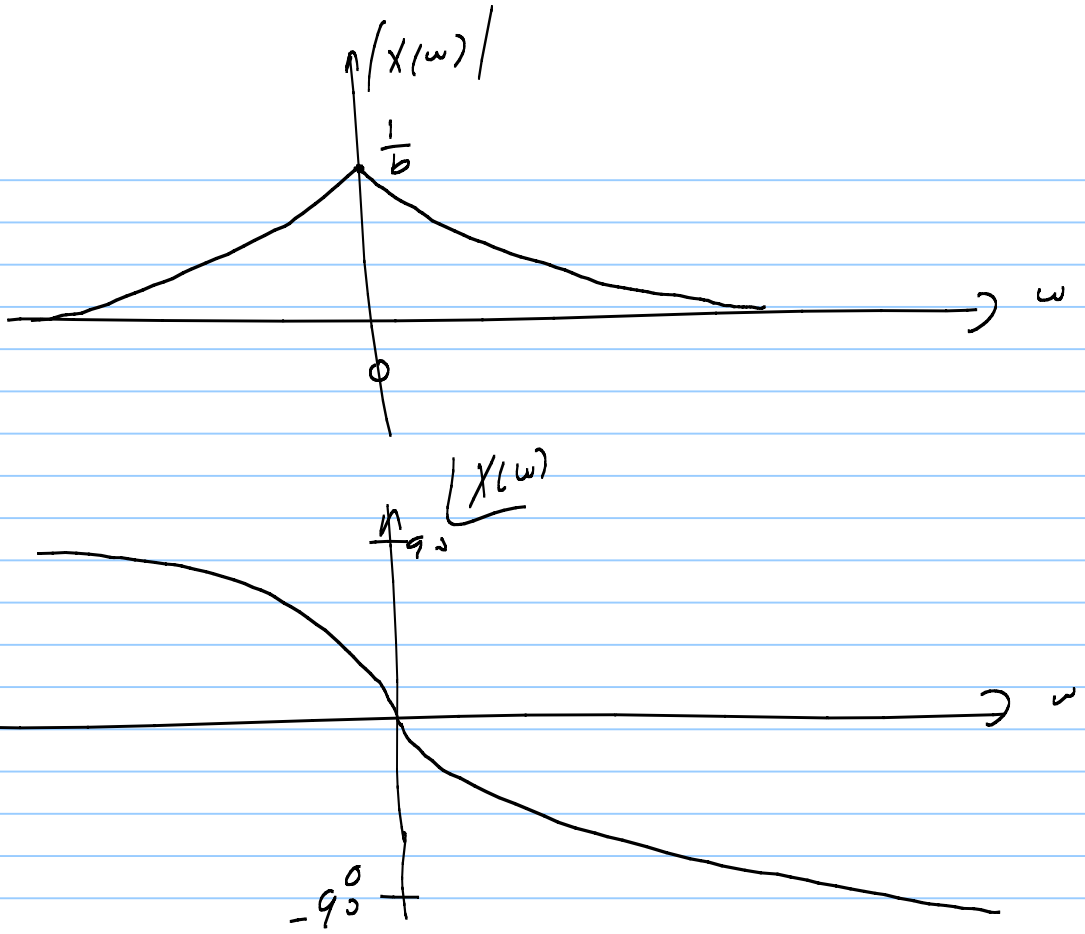
$$X(\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$\therefore X(\omega)=\int_{0}^{\infty} e^{-bt}e^{-j\omega t}dt=\int_{0}^{\infty} e^{-(b+j\omega)t}dt$$

$$=\frac{1}{-(b+j\omega)}e^{-(b+j\omega)t}\Big|_{0}^{\infty}=\frac{1}{-(b+j\omega)}\left[0-1\right]$$

$$\therefore \boxed{X(\omega)=\frac{1}{b+j\omega}}$$

$$|X(\omega)|=\frac{1}{\sqrt{b^2+\omega^2}} \qquad \angle X(\omega)=0-\tan^{-1}\left(\frac{\omega}{b}\right)$$


|X(ω)|
1/b
ω
∠X(ω)
90°
-90°
ω